Clerselier to Fermat

Saturday, May 13, 1662

Sir,

1. It is by order of the Assembly which is held weekly at M. de Montmort's house, that I write you today to honorably apologize for a mean-spirited Latin word that I put in the letter that I gave myself the honor of writing to you eight days ago, which I read to him last Tuesday. That is the only thing that the Assembly found it necessary to restate, and I indeed felt it myself when I wrote it: also I have tried to tone it down by the preceding corrective. However, notwithstanding that, I was publicly reprimanded, and just as soon as I proposed to apologize to you by the first ordinary post, which I do today much more freely since additionally by this submission I will acquaint you with the ingenuity of my procedure, which will give me another opportunity to tell you something that I am obliged to make as a response to some objections made to me by some in the Assembly, in order to make the thought of M. Descartes on refraction clearer by a familiar example which is altogether proper to the subject.

If I had not been so impatient as to send you a thing that was ready for more than fifteen days and if the engagements that I had did not require me to see M. de la Chambre,[1] I would have avoided the reproach of the Company and would not have made this mistake. But I was afraid that it would be necessary for me to wait still longer to speak about it at the Assembly, which had already twice put off the reading that I wished to make to them, since they wished to hear at the same time the thoughts of M. Petit, who let them know as soon as your letter arrived before them, that he had several things to say against both what you have written to M. de la Chambre, and against what M. Descartes has written.

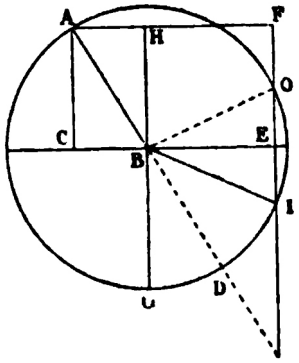
[1] *de faire voir dès-lors à M. de la Chambre*

For myself, I, who was not present at the Assembly when your letter was read there the first time, and frequently exempted myself from being there, because of some more important matters, such as the detention of M. de la Haye, my son-in-law, which caused me to plead for his freedom at the court, that I had no sooner seen your letter than I felt obliged to respond, it being the latest in a series of small quarrels that we had already had on the same matter, and also because you do me the honor of naming me three times in your letter and seemed to be inviting me to reply.

I had therefore prepared my response just as quickly as I could, and I thought to show it to the Company, but they did not think that would be appropriate, since they did not want to anticipate M. Petit in the retort that he had promised to give you. But, fearing that this would take too long, last Saturday I decided on my own to send it to you before having it seen by the Company, whose advice I received too late to prevent me from making this mistake, but not too late to prevent me from apologizing to you and begging your pardon.

And in order that I may in some respect deserve your forgiveness, permit me to explain once more at length what I did not explain the last time, to make you understand what I think of the thoughts that M. Descartes had on refraction.

Fig. 102.



2. It is certain that if ray AB be considered by itself, such as when it is in the air, it goes neither from left to right, nor from above to below, but its entire tendency is to go towards D and has only a single direction. But as soon as it is opposed by another medium, CBE for example, into which it must enter, it can be said and it is true that in regard to this medium it has

different tendencies. For if it were opposed directly, its fall is perpendicular and has only a direction in this respect: but if it is opposed obliquely, as it is on page 17 of the *Dioptrics* (*fig.* 102), then in regard to this medium the ray has a double direction: one which brings it towards it, which is from above to below; and the other which carries it from left to right, to which direction the medium is not at all opposed.

And if it had opposed it in another way, the same direction, which is now from left to right, could be that which carries it towards the medium, and the other could be that to which the medium is not at all opposed. And accordingly as this medium is more or less inclined to the ray, the two tendencies or directions that the ray has with respect to it are different and can have different proportions between each other.

3. But when I speak of tendency, direction, or determination, you will not imagine that I mean to speak of a direction without force and without movement, which would be chimerical and impossible, since there cannot be direction without motion or effort; but when I speak of direction or determination towards some place, I mean the entire portion of the motion which is determined to go towards that location.

Therefore, accordingly as the medium is more or less inclined to the ray, the force which, with respect to it, carries it towards a given location can be greater or lesser than that which carries it towards the other. For example, if the angle ABC is equal to angle ABH, then the two parts of the motion, one of which carries it below and the other to the right, are equal; if it is less, then its force is less; and if it is greater, the force is greater.

4. But, whatever be the inclination of the ray upon the medium, there is always a portion of the force of its motion to which this medium is opposed and another to which it is not. Yet, so long as the ray is in the air, the proportion, whatever it may be, which holds between these two portions of the motion that we assume uniform, carries it along line AB and, so long as nothing changes it or so long as they change while keeping the same proportion between them, the ray goes always along a straight line.

But when the ray AB on page 17 meets another medium at point B, if this medium is not as easily penetrated by the ray as was the air, then the path of the ray must change, because this medium is only opposed to the determination or to the part of the motion which carries it towards itself, and not at all to the other, which at the point of contact remains precisely the

same. However, since there is no longer the same proportion between these two parts of motion which, together, carried the ray along line AB before, they must change its determination and carry it towards the point where the direction tends which is adjusted to the proportion which is then found between them, and thus to draw it away from the perpendicular.

If, on the contrary, the medium which is opposed to ray AB presents more difficulty to its passage than did the air, this new ease that it brings and which is only experienced by the part of motion which tends towards it, and not by the other, as I have already said, must change its direction, because this changes the proportion between the two parts of which the entire motion of the ball is composed, and consequently turns it away towards the perpendicular, which happens when a ray of light passes from air into water or into glass.

5. And to facilitate the comprehension of all this by an easy example, imagine a spherical body, quite hard and well polished, set upon a very stiff and well polished board whose end rests upon the extremity of the table, such that the board be inclined upon the table and make an equal angle with it. It is certain that this moving body will roll on this board, and this more or less quickly as the board will be more or less inclined to the table. But, whatever may be the motion of the body on this board, it is certain that with regard to the table it has two determinations: one which draws it towards it, by which it descends; and the other which carries it towards one of the walls of the room, by which it advances towards that side.

And it is so true that it has these two impressions, that it still keeps both of them when it is in the air off of the board; and if only one remains to it when it is above the board, it would only follow that one; for example, it would fall perpendicularly to the earth as soon as it left the board, if only the impression of its fall remained.

But consider what will happen to the moving body when it is at the point at which it leaves the board, and you will see that the same thing happens to light when it passes from air into water. And since then the part of the movement which carries the moving body downwards finds more ease or less resistance to its action, when it is above the board and in the air, than it had when it was on the board; and that which carries it towards the wall remains the same (even though it is still the same total force which pushes the moving body at this point, and though the force of the two parts of its motion taken separately be the same), nevertheless since the proportion

which existed earlier between the ease or the resistance that the medium presented to these two forces has changed, and since at this point of exit it finds it easier to descend than before, although it finds neither more nor less ease in moving towards the wall, it therefore happens that it no longer follows the direction of the line that it had traversed on the board, but that it will take another, which is proportional to the increased ease which is found in one of its forces than the other. This causes the movement, upon leaving the board, to approach the perpendicular, as light also does upon entering water, for the same reason.

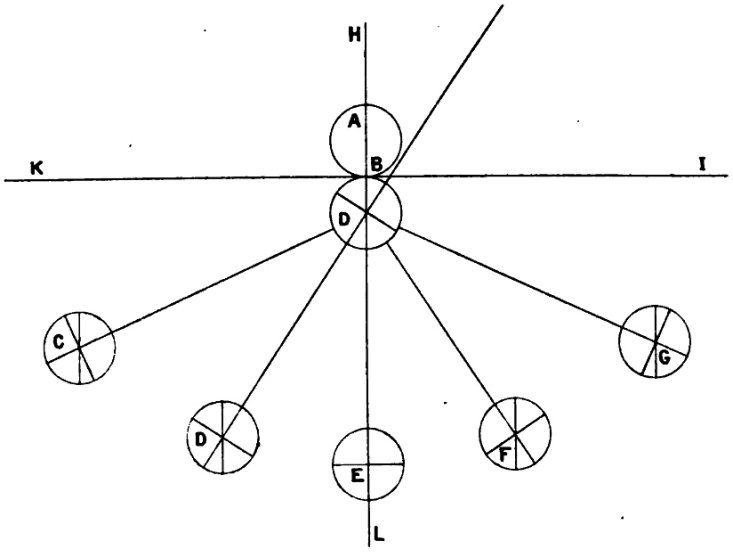
And this is, in my view, one of the easiest possible things to conceive, and it is also, in my opinion, all that M. Descartes wished to say on the subject of refraction. I do not claim, however, to have persuaded you with this: it is enough that I have made myself heard, so that you do not think that I follow M. Descartes blindly or that I contradict you to entertain myself. I resemble you in this point – that I love and seek out only the truth, and this similarity that I have with you makes me hope that you will not repudiate me, while I everywhere avow, etc.

P.S. To further clarify this matter, I will also bring up here an example which to my thinking resolves the majority of the difficulties that may be brought up regarding what M. Descartes has said concerning refraction in his *Dioptrics*.

It is certain from experience that, in whatever manner ball A be impelled towards point B by balls C, D, E, F, G, whatever be the different determinations we may assume their routes be composed of, they will always push it towards B (*fig.* 103).

First, for ball E, it is clear that it must push it towards H, since ball A completely opposes its determination; but that which is clear for ball E must similarly be understood for the others, which, although they come towards ball A obliquely, only touching it at point B and only pushing it insofar as they descend towards H, and not insofar as they move towards I (or towards K). This is why they would not be able to impress any movement upon this ball, other than to move it towards H. Yet, although the determinations of balls D and F be opposed, insofar as one goes to the right and the other to the left, they are not at all opposed insofar as they descend and then they must produce the same effect on ball A, which is to impel it towards H.

Fig. 103.



But, if we assume that the ball A be hard and immobile, all these balls, after having come into contact with it, will have to change the determination that they had to move towards H into that determination of moving or reflecting towards L, and to keep the other determination if they had any, which the ball cannot change, because it is not opposed in that direction: and this explains reflection at equal angles.

If we assume that these balls have communicated their movement to ball A, this can only be in the direction that it is opposed to them, and therefore only the motion towards H can be changed, and not that towards I (or towards K), which must consequently remain the same in its entirety. If even though these balls lose at point B the force which determines their motion towards H and lose nothing of that which determines them to move towards I, they are made to move away and to take another direction at this moment, which they always maintain, no matter what resistance the medium brings after that, which could indeed cause them to move more slowly but could not change their direction, because it may very well be opposed to their speed, but not to the direction that they have taken, since we assume that it is equally easy or difficult to enter into or penetrate in all directions. And that explains refraction which moves away from the perpendicular.

If on the contrary we assume that, the balls arriving at point B, ball A gives in to them more easily and draws them away, so to speak, towards H, making the balls descend more quickly, but that does not change their

movement towards the right (or towards the left) to which it is not at all opposed. At the moment that they are at pint B the balls are more disposed to go towards H than they were before, and are neither more nor less disposed than they were before to go towards I. And thus these balls must change their direction, and must maintain this new direction, once they take it. And that explains refraction towards the perpendicular.

And to make it clear that the greater or lesser resistance of the body of the medium has no effect on it, and does not change the determination that the ball gets at B, we will consider what could happen to the ball A according to the different cases that could be imagined. For example, if the ball E falls perpendicularly upon A and if it communicates to it half of its motion, where will it go? Without a doubt, it will go towards H, and the force that it receives at that moment can only determine it to go in that direction: but is that to say that in going towards H it will trace out in two moments a line just as long as E made in one moment? Yes, without a doubt, if you assume that the medium that it traverses makes way for it just as easily as the other did; but if this medium resists it more, it would describe a shorter one, just as it can describe one equal in length or even a longer one, if this medium resists the force that it receives in an equal amount or less.

If we assume that one of the other balls C, D, F, G collides with A at point B, the same thing will follow, namely that because of the force that it will receive, it will have to take its determination towards H as before, at the very moment that it is touched. And the quality of the medium will not change this determination, if not because of receiving less force, then because only being encountered obliquely it is not impelled by all of the force of the ball which touches it, it will go less quickly.

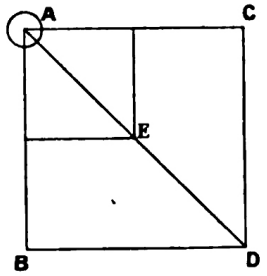
If we assume ball A to already be in motion towards I, the fall of one of these balls upon it will bring no change to the determination that it had to go in that direction, that is to say the entirety of the force of its movement which determined it to go towards I, and therefore it must continue to go towards it as it did before. But it must also at the same time move in the same direction that is determined by the impression that it has just now received by the fall of one of these balls, even if after this moment it must take its direction.

But if we assume that the medium in which it is found after this, resists it more than the other one did, that does not change the determination that it has taken, but only causes it to travel slower than it would have. For finally the proportion which held at that moment between these two forces

gave it a determination to go somewhere, and whatever ease or difficulty the body encounters in the medium that it must traverse, since it is equal in all directions, it cannot change anything of the determination that it took at the surface, and must not deflect it more or less.

And the same proportion is maintained here between strong or weak movements that are equally proportioned. For example, let ball A be impelled by two equal forces towards B and towards C at the same time, what must happen to it, if it is in the air? These two forces, having a great effect on it, will impel it to D in one moment. But if it were in the water, then these two forces, not having such a great effect on it, would only impel it to E, but for all that would not change the direction.

Fig. 104.



And what I say about ball A, which is impelled by these equal forces in two different media, must be understood just the same for any other sort of proportion which may hold between two forces: that is, the difference of the medium does not change the direction towards which the forces that it has that determine it at the first moment, but only change its speed.

For example, let ball A be impelled at the same time by two forces, one of which impels it twice as strongly towards C as the other does towards B. What must occur if the ball is in the air? What will happen, is that these forces, having a great effect on it, will impel it in one moment of time to D. But, if it were in water, then these two forces would not have as strong an effect on it, but would continue to act on it in the same proportion as before in the two directions, since water yields equally in all directions, and would therefore impel it only as far as E; but it will not in any way change the direction that it took on at the first moment.

And thus, in regard to the first assumptions that M. Descartes makes, when he makes use of the example of a ball to explain reflection and refraction in the second chapter of the *Dioptrics,* assuming that neither the weight

nor the lightness, the size, nor the shape, nor any other such foreign cause changes its course,[2] what he says afterwards is true: to wit, that only the determination that the ball takes at the moment it is at point B need be considered, without worrying about what change its speed may undergo in the medium that it then traverses, since it is only at point B that it must change its direction, because of the change which occurs at this point in the proportion which is between the two forces which together compose its entire movement; and the direction that it has taken at B will be maintained afterwards, and it moves in that direction more or less quickly to the degree that it meets more or less resistance from the medium.

[2]Apparently, any properties that an actual ball might have are off-limits.