Fermat to Clerselier

Sunday, June 2, 1658

Sir,

1. I am so impassioned for the glory of M. Descartes that nothing you could do would make me happier than countering the opinions of the skeptic who opposes his thoughts. But take care, Sir, that it is relevant to leading your work to its end, and take care to entirely turn against their authors all that you call either paralogisms or sophistries. It is not enough to say that the meaning of M. Descartes has been ill-understood by those who took it up; it must be proven that the explanation that you give goes directly and without detour to his conclusion, and finally that his proof is demonstrative.

2. I was under the impression that the ball that maintains its direction and its route does not lose its determination, and it was with reason that I inferred this from the difference that M. Descartes establishes between motion and determination. But, without pressing myself more to prove the consequence that I would draw from his reasoning, I consider myself sufficiently informed of his thoughts and of yours, which call for the fact that "the determination of a moving body should be considered changed, not only when it leaves the line along which it was previously moving, or when it moves in the opposite direction along the same line, but also by moving in the same direction along the same straight line, provided that it be less far than it was previously determined to go in that direction."

"And it is in this third manner," you say, "that the quantity of the determination of the ball has decreased as much as the motion," when it moves along line HBG on page 17 of the *Dioptrics*.[1] But take care that this does

[1]See *fig.* 56 here.

not fall into a *petitio principii*.[2]

Therefore you imply, on page 17, that the cloth not being *at all* opposed to the determination of left to right, these words mean that this determination goes forward as much towards the right as it had earlier. It is this that I deny and this which must be proven: for, although the cloth does not *prevent* the ball's continued progress towards the right, it nevertheless may progress in this direction slower or faster than before. Yet, only from this — that the cloth does not prevent the progress towards the right — you infer that this progress must be *exactly* the same, that is to say neither more nor less quickly than before. It is therefore $\alpha\acute{\iota}$'$\tau\eta\mu\alpha$ $\alpha\iota$'$\tau\acute{\eta}\mu\alpha\tau o\varsigma$,[3] and only one of these two must be true: either the *medium*[4] is the same as the conclusion, or the conclusion has been wrongly derived.

Perhaps you will say that the words "not at all" hold all the mystery, and that by saying that the cloth is *not at all* opposed in this direction, everything else is easily deduced. But it is necessary still to return to this: if by the phrase "not at all" you mean that the cloth does not prevent the ball from continuing its progress towards the right and that its progress is made equally in equal times, I deny it and this is what must be proven. If you mean that the cloth is *not at all* opposed, that is to say that it does not prevent the ball from continuing to advance towards the right, then without making further assurance that its progress must continue to be made in equal times, your conclusion will always be disputable.

Whence it clearly follows that M. Descartes wished to give words for things, and that when treating two different propositions on the subjects of reflection and refraction, he wanted to accommodate his reasoning to the first which he knew and to the second about which he had perhaps too quickly reached a conclusion.

3. It is not, as I have often challenged you, that his proportion of refractions *could not* be true; but I had to tell you that I do not at all consider it to have been proven, and that in any case you show too much kindness in making believe that you approve of my thinking on this same subject, since, if what I have written to M. de la Chambre is true, then what M. Descartes

[2]Begging the question.

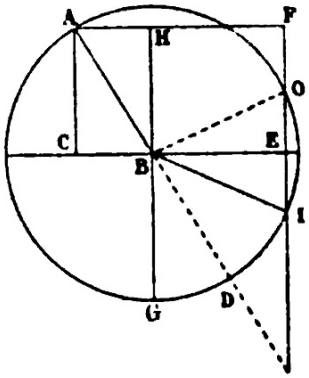
[3]This Aristotle's Greek, with the same meaning as the *petitio principii* in the preceding paragraph.

[4]A triple pun: the *means* of arriving at his conclusion, the *medium* through which light travels, and the fallacious *middle term* of a *petitio principii*.

thinks he has demonstrated is necessarily false, for these two opinions are altogether contradictory and incompatible.

But let us assume, if it is possible, that the proposition of M. Descartes were true. We must at least make sure that nothing goes wrong in what follows, and it is to the friends of the deceased to anticipate all the cases which could cause difficulties for the supposed verity of this proposition. Here is one, for example, that you must try to resolve.

Fig. 56.



Suppose, on page 17, that the ball encounters, instead of the cloth or water, a hard and impenetrable body, and that, when the ball arrives at point B, it loses half of its speed. For this supposition is possible and, although the body CBE contributes nothing to the diminution of said speed (as it must in the example of M. Descartes, when it is cloth or water), nevertheless we can imagine and assume that when the ball arrives at point B, it loses exactly half of its speed, without worrying about whence this diminution comes, since the same M. Descartes, on page 20, assumes or imagines at point B a new power which augments the movement or the speed of the ball:[5] such that I do not believe that the friends of M. Descartes would be so unjust as to deny that this supposition can be not only imagined, but put into application.

Having assumed this, we need only transport the above reasoning of M. Descartes above the plane, and we will be able to say with him that, to know the path that the ball must take, we must consider that its movement differs entirely from its determination to move in one direction rather than another: whence it follows that their quantities must be examined separately.

[5]Descartes actually creates the scene of an imaginary tennis racket hitting the ball downwards just as it enters the water! See his *Dioptrics*.

Let us also consider that, of the two parts of which one can imagine this determination to be composed, it is only that which causes the ball to move from high to low that can be changed by the encounter with the plane CBE, and that, as for the one which causes it to tend towards the right, it must always remain the same as it had been, because the plane is not in any way opposed to this direction.

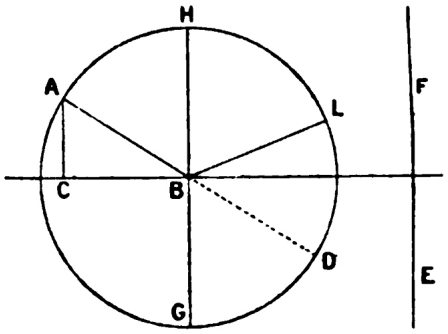
And then, having described around center B the circle AFD and having drawn at right angles to CBE the three straight lines AC, HB, FE, such that there is twice as much distance between FE and HB as between HB and AC, we will see that this ball must tend towards the point of the circle where the line FE intersects the circle above the plane; this point can be designated by the letter O.

For, since the ball loses half of its speed in its encounter with the plane at point B and since by assumption it may not pass through it, it must take twice as much time to pass above from B to some point of the circumference of the circle AFD, as it took to move from A to B. And, since it loses nothing at all of the determination that it had of advancing towards the right side, in twice the time that it took in passing from line AC to HB, it must make twice as much headway towards this same side, and consequently arrive at some point on the straight line FE at the same instant that it arrives at some point on the circumference of circle AFD. The which would be impossible if it did not go towards O, inasmuch as it is the only point above the plane CBE where the circle AFD and the straight line FE intersect.

If this reasoning, which is just the same as that of M. Descartes, merely being transposed, is not conclusive, why, for goodness' sake, would the one of M. Descartes be conclusive? Does the demonstration below become a paralogism above? I do not believe that you would be of this sentiment, or that you wished to attribute everything to the sole name, as it were, of M. Descartes, and to his inspiration.

4. This being so, let us go on to the figure on page 19 (*fig.* 89), and let us likewise suppose that the plane CB is a hard and impenetrable body, and that the ball, arriving at point B, reduces its speed such that the line FE, being drawn as in the preceding example, does not cut the circle AD.

Fig. 89.



By assumption, this ball, cannot penetrate below the plane. Neither can it reflect at equal angles, for its determination towards the right would not be the same. Finally, whatever angle you took for reflection above the plane, its progress towards the right would always be less than before. In truth, were you to make it roll on the diameter CB in a straight line, its determination towards the right would change again, as can be seen by the eye and as can be clearly deduced from the assumption: for it would be necessary that at the same time that the ball arrives at some point on the circumference, it would have arrived at some point on the line FE, which is impossible.

What then, will become of this ball? It is for you, Sir, and for the friends of M. Descartes, to furnish it with a passport and to mark out the path by which it can leave this fatal point. I would say more if I did not fear to pass into your thoughts as a man who would desire to

Barbam vellere mortuo leoni.[6]

I await, Sir, your reply or that of M. Rohault, who I esteem as I must; and I assure you in advance that I seek only the truth without cavil, and that I am with all my heart, Sir, your very humble and very affectionate servant,

Fermat.

[6] "Pull the whiskers from the dead lion." Martial, book 10, epigram 90.