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Research Review

The Beginnings of a Deterministic Theory of Turbulence

S. Bardwell

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Editorial

This issue of the *International Journal of Fusion Energy* contains the first article to appear here on the engineering frontiers of fusion research. Coauthored by Bruno Coppi, one of the foremost plasma physicists in the world today, whose work is unique for its geographical scope as well as for the way in which it encompasses both the engineering and theoretical aspects of fusion, this paper is an exciting introduction to the forefront of the practical problems in fusion research and the development of a commercial power reactor.

The second article in this issue is a research review, covering the Sept. 1979 workshop in LaJolla, California, on structured states in turbulent fluids, which was attended by two directors of the Fusion Energy Foundation.

This is the last issue of the *IJFE* to appear in the present quarterly format; beginning with the next issue, the redesigned journal will appear semiannually as a much more extensive publication, with room for several longer articles. The editors look forward to this greater freedom to publish significant works and translations of historical breakthroughs in full. An annual review of engineering and theoretical progress in fusion physics will now become a regular *IJFE* feature. Each issue of the new *IJFE* will thus replace two issues of the shorter journal for subscription purposes.

Authors wishing to submit manuscripts for publication in IJFE *should send two (double-spaced) copies of their work with stats of all figures to the Managing Editor,* The International Journal of Fusion Energy, *Fusion Energy Foundation, 250 West 57th Street, Suite 1711, New York, N.Y. 10019.*

Thermonuclear Ignition By Means of Compact Devices[*]

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The peaceful use of nuclear fusion energy occupies an important place among the objectives that science has been pursuing since the 1950s. Realizing this objective would constitute a permanent solution to the world's energy problems. In fact, in comparison to the exploitation of nuclear fission energy, which was realized at an economical level many years ago, the development of fusion energy will present a considerably smaller degree of risk and will use a low-cost, practically inexhaustible, easily accessible fuel reserve in the form of deuterium obtained from seawater.

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For a nuclear fusion reactor it is necessary to bring a mixture of hydrogen isotopes to a certain critical temperature and keep it confined for a sufficiently long period of time. Since the relevant temperature for a fusion reactor is on the order of tens of millions of degrees Celsius, the fuel mixture will be in the plasma state; that is, its atoms are split into ions and free electrons. Thermonuclear fusion can be achieved and maintained with the mixture at a low density (10^{14}–10^{15} particles/cm^3) and confined in a high magnetic field (*magnetic confinement*), or it can be achieved with a denser mixture (10^{22} particles/cm^3) that is brought to fusion conditions by interaction with beams of laser light or high-energy particles (*inertial confinement*).

In this article we shall discuss some of the problems involved in magnetic-confinement nuclear fusion devices, particularly those devices that use reduced dimensions and high magnetic fields (higher than 100 kG) to realize "thermonuclear" plasmas; that is, plasmas in which the fusion reaction plays a fundamental role. The experimental possibilities of such devices would be significant in developing a prototype fusion reactor.

THE IDEAL IGNITION TEMPERATURE

The nuclear fusion reaction attainable in the first magnetic-confinement reactor is the result of an interaction between a nucleus of deuterium and a nucleus of tritium, with energy sufficiently high to overcome the Coulomb repulsion of the nuclei. The products of this reaction are a neutron with energy above 14 MeV and an alpha particle (helium nucleus) with an energy higher than 3.5 MeV:

$$\mathrm{D} + \mathrm{T} \rightarrow \underset{(14.06\ \mathrm{MeV})}{\mathrm{n}} + \underset{(3.52\ \mathrm{MeV})}{\alpha} \tag{1}$$

The plasma here would be confined in a toroidal magnetic configuration of the tokamak type, as described in Refs. 1–5.

Figure 1 shows the process of the energy loss from the alpha particles produced in the D–T reaction in a tokamak-type system, as a function of their energy. Note that the alpha particles initially lose energy predominantly through collision with the electrons and begin to lose energy in appreciable amounts to the ions only when their energy has decreased to levels around 0.5 MeV.

If the alpha particles are produced in relevant quantities and are confined inside the plasma during their deceleration, the energy they deposit in the plasma makes an important contribution in compensating for the various energy losses of the plasma itself. In an actual working

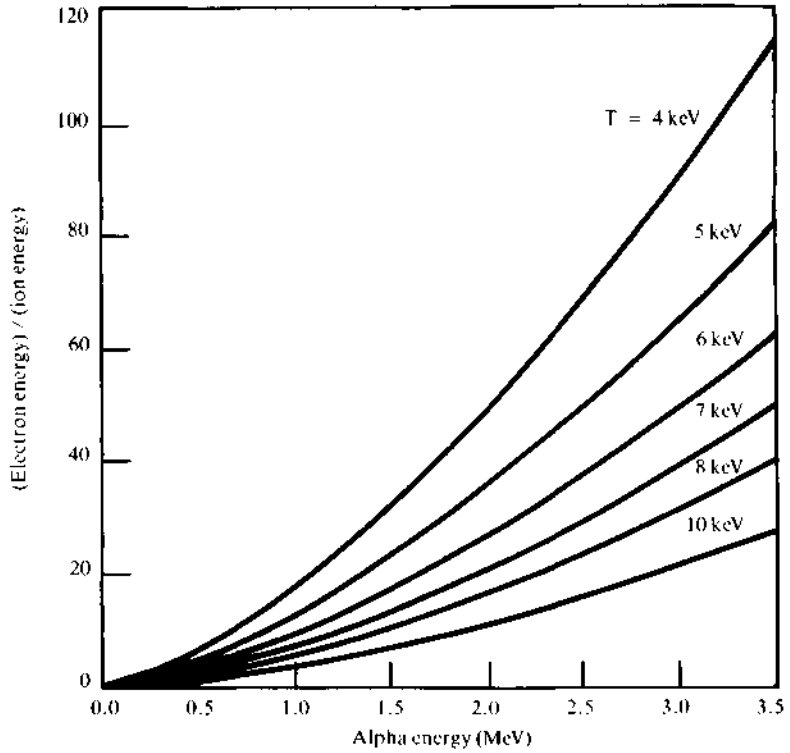

Figure 1. Alpha-particle energy loss. The curves show the relationship between the energy lost by the alpha particles to the electrons and that lost to the ions as a function of the energy of the alpha particles for various temperatures of the plasma.

fusion reactor the energy deposited by the alpha particles will compensate for all the plasma's energy losses (*actual ignition*).

The number of alpha particles generated per cubic centimeter per second can be expressed by the equation

$$(\dot{n}\alpha)_{\text{generated}} = n_D n_T \langle \sigma v \rangle, \tag{2}$$

where n_D and n_T are the densities (particles per cubic centimeter) of the deuterons and tritons, respectively; σ is the cross section for the fusion reaction; v is the relative velocity of the interacting nuclei; and the angle brackets indicate the average over the distribution function of the deuterons and tritons. If we generally suppose that such functions can be approximated by Maxwellian distributions, we have

$$\langle \sigma v \rangle \simeq 3.68 \times 10^{-12} T^{-2/3} \exp(-19.94 T^{-1/3}) \ \mathrm{cm}^3/\mathrm{sec},$$

where T is the temperature of the plasma in kiloelectron volts.

The power per cubic centimeter generated by the fusion reactions can be expressed by the relation

$$P_\alpha = n_\mathrm{D} n_\mathrm{T} \langle \sigma v \rangle Q_\alpha, \tag{3}$$

where $Q_\alpha = 3.52$ MeV is the energy of the alpha particles at the moment of their generation.

For a plasma temperature of 10 keV, we have

$$P_\alpha \simeq 6 \times 10^{-29} n_\mathrm{D} n_\mathrm{T} \ \mathrm{W/cm^3}. \tag{4}$$

For a plasma of relatively high density ($n > 10^{14}$ particles/cm^3), one of the principal energy losses is through the emission of bremsstrahlung, which is produced when an electron collides with a nucleus of deuterium or tritium and undergoes a change in the direction of its motion.

The equation for the power per cubic centimeter radiated by the bremsstrahlung, which is valid for a deuterium and tritium plasma, is

$$P_\mathrm{brem} \simeq 2.14 \times 10^{-30} n_\mathrm{D} n_\mathrm{T} T_e^{1/2} \ \mathrm{W/cm^3}, \tag{5}$$

where T_e is the electron temperature in kiloelectron volts. It should be noted that P_α and P_brem show the same dependence on the density of the components of the mixture.

The conditions under which the energy deposited in the plasma by the alpha particles equals the energy lost by bremsstrahlung are established at a temperature greater than 4.2 keV (about 5×10^7 °C), which is the given ideal ignition temperature (see Fig. 2). As the temperature increases, the rate of fusion reactions increases more rapidly than the rate of the bremsstrahlung emissions; in the absence of other types of thermal energy losses, the temperature would continue to rise.

Although the effective ignition of the plasma occurs at a temperature higher and less easy to define (because the mechanisms of energy loss in a plasma are not well known), the achievement of an ideal ignition temperature in a D–T plasma that may have a sufficiently long energy-confinement time ($n\tau \geq 10^{14}$, as required in a reactor) is a fundamental objective. In fact, heating of the alpha particles in such a plasma would play an important role, and various phenomena that are difficult to predict theoretically could be studied experimentally. There-

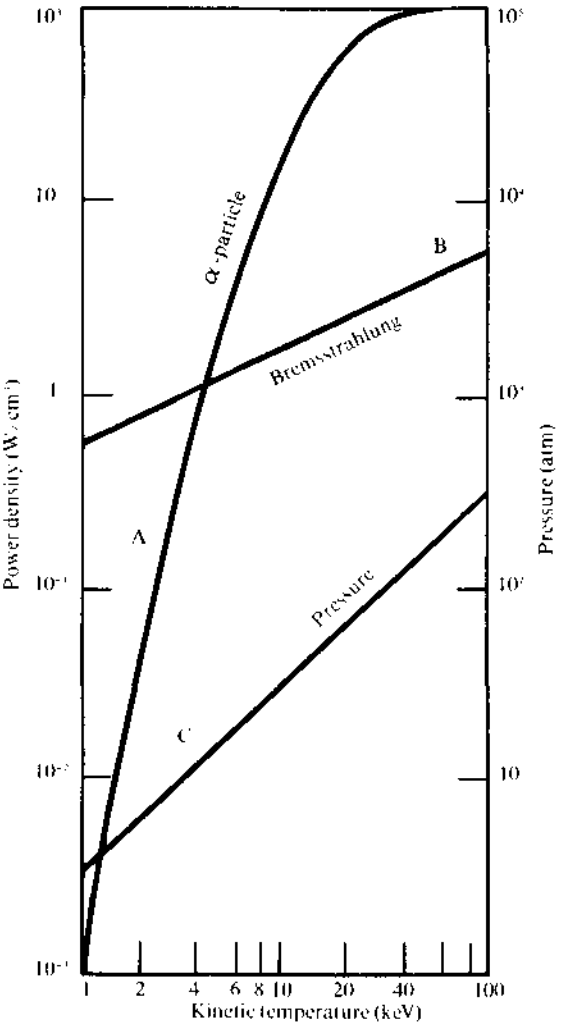

Figure 2. Curve A: Power density associated with the alpha particles produced from a D-T fusion reaction as a function of temperature for a plasma with a density of 10^{15} particles/cm^3. Curve B: Power density radiated from the plasma by emission of bremsstrahlung. The intersection of curve A and curve B indicates the so-called ideal ignition temperature. We note that the corresponding plasma pressure, indicated in the curve C, is around 15 atm, where the mean magnetic pressure necessary for confinement is on the order of 900 atm.

fore, in order to study truly "thermonuclear" plasmas as soon as possible, it is important to develop such a device for deuterium–tritium experiments with the goal of achieving the ideal ignition temperature, with $n\tau \geq 10^{14}$.

In the case of a magnetically confined plasma in a toroidal configuration, confinement of the alpha particles requires that the radial amplitude of their orbits (one type is shown in Fig. 3) be smaller than the minor radius of the plasma. Considerations based on analytical formulas approximated for the representation of the alpha-particle orbits as well as numerical calculations have allowed detailed descriptions of the great variety of orbits possible and have demonstrated that the conditions for confinement of more than 95% of the alpha particles are given, in a reasonably accurate manner, by

$$I \geq 9/A \ \text{MA}, \tag{6}$$

where I is the toroidal current and the "aspect ratio" is

$$A \equiv R/a,$$

where R is the major radius and a is the minor radius of the torus.

For values of $A \sim 2.5$–3, therefore, we must have

$$I \gtrsim 3\text{–}3.5 \ \text{MA}. \tag{7}$$

For considerations of stability of the plasma column as well as for questions of technical feasibility, on the other hand, the plasma current cannot be increased arbitrarily. In particular, the so-called safety factor q_s must be considered:

$$q_s = 5aB_T/AI = 5a^2B_T/RI, \tag{8}$$

where I is the plasma current given in amperes, B_t is the toroidal field given in gauss, and a and R are given in centimeters. This factor must be $\gtrsim 2$ to avoid strong instabilities in the plasma.

From Eq. (8) we see that to obtain high currents we must have large values of the quantity a^2B_t/R. It is interesting to note that using $B_T = I_c/5R$, where I_c is the total current in the magnetic coil for the toroidal field, and Eq. (8) we get

$$I_c = IA^2q_s. \tag{9}$$

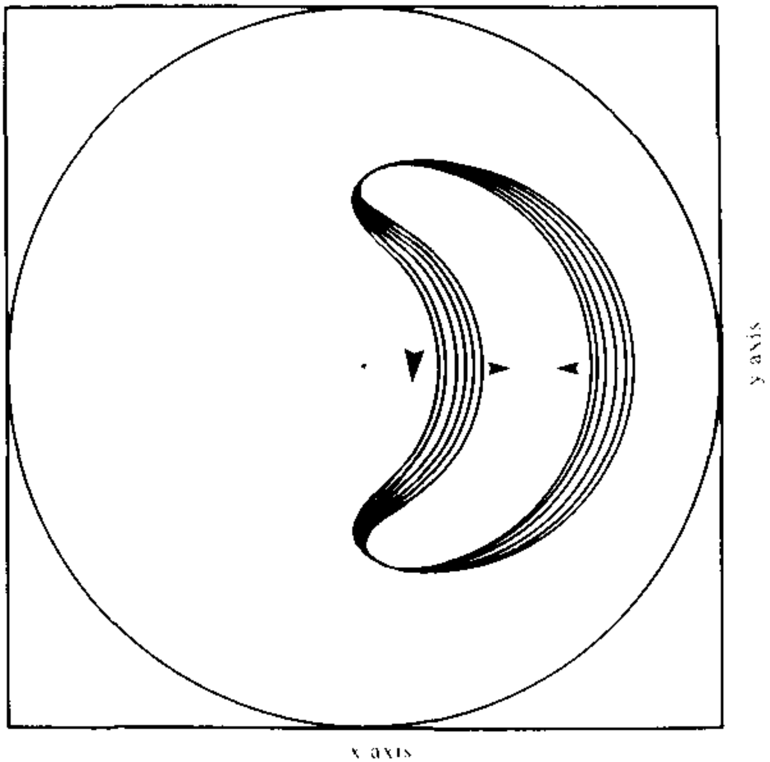

Figure 3. Projection on a meridian plane of the guiding center orbit of an alpha particle during its deceleration in a toroidal plasma. The motion of the alpha particle consists of the movement of the guiding center and a gyration around the guiding center. The horizontal arrows indicate that the length of the orbit in a "banana" shape is restrained during deceleration; the vertical arrow gives the sense of the banana orbit.

We can thus minimize I_c and, accordingly, the dimensions required by the toroidal magnet, thus reducing q_s and A. With $q_s \simeq 2$ and $A \simeq 2.5$,

$$I_c \simeq 40 \text{ MA}.$$

Such a current is easily attainable in compact toroidal devices where R is less than 1 m, which are also interesting because of their lower cost in terms of producing plasmas in which alpha particles are confined.

Another interesting possibility in such devices is that of obtaining desired plasmas by minimizing or obviating the need for nonohmic supplementary heating of the plasma.

DESCRIPTION OF AN "IGNITOR-TYPE" MAGNETIC-CONFINEMENT SYSTEM

Let us examine a type of tokamak called the Ignitor or Alphator. As has been noted, the plasma column is magnetically confined in a toroidal configuration with axial symmetry. The magnetic field necessary to maintain the plasma equilibrium in this configuration is produced partly by electrical currents circulating in an external metal conductor (*toroidal* field and vertical field) and partly by currents induced in the plasma itself (*poloidal* magnetic field)—see Figure 4. The current flowing in the plasma, which is in practice the secondary current of a transformer, also furnishes a natural means of heating (ohmic heating) by virtue of the resistivity of the plasma itself (Joule effect).

A plasma is a conducting medium whose resistivity associated with the transfer of momentum of the electrons acquired from the induced electric field to the ions is small but not negligible. For example, at an electron temperature of about 10^7 °C, the resistivity of the plasma is about the same as that of copper at atmospheric temperature, but decreases as the electron temperature increases, according to the following expression:

$$\eta = 2.5 \times 10^{-6} Z T_e^{-3/2}\ \Omega\text{-cm}, \tag{10}$$

where Z is the ionic charge and T_e is the electron temperature in kiloelectron volts. This behavior, so different from that observed in metallic conductors, arises from the fact that the frequency of ion–electron collisions by Coulomb interaction decreases rapidly (as v^{-4}) with an increase in the relative velocity v of the interacting particles.

The power density produced by the ohmic heating is related to the resistivity of the plasma and to the current density J by the expression

$$P_J = \eta J^2. \tag{11}$$

This is an important result for the realization of confinement configurations in which relatively elevated current densities can be induced.

As can be seen from Eq. (8), which can be written in the form

$$J \simeq 5B_T/\pi q_s R,$$

in order to achieve high values of the current density at a fixed value of

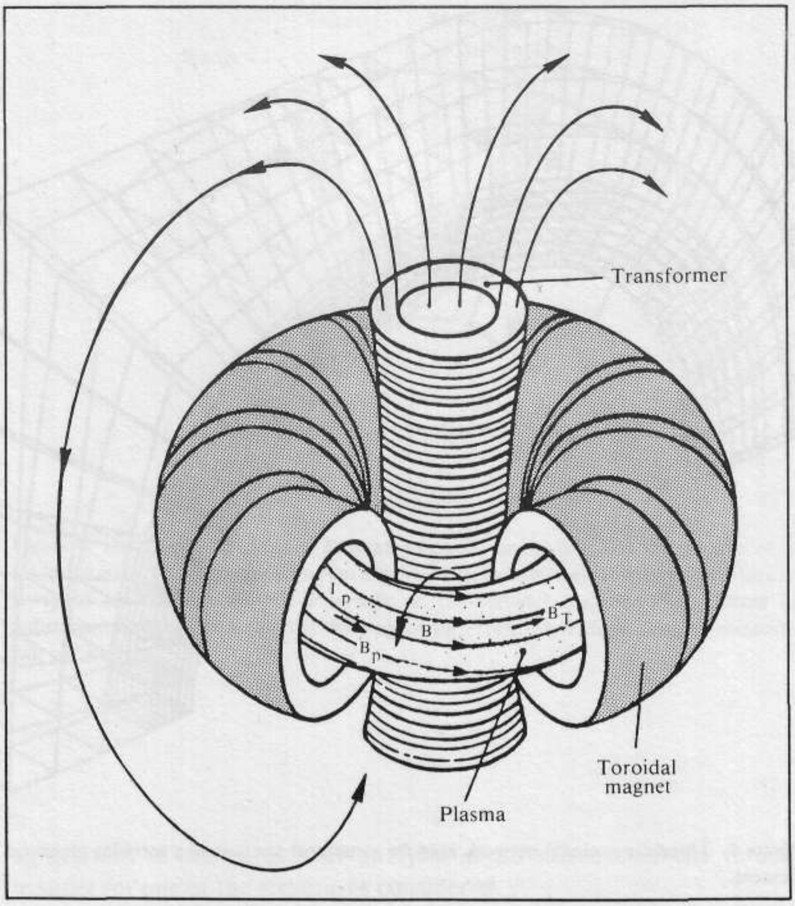

Figure 4. Representation of magnetic fields present in a tokamak configuration. The plasma current I_p, the toroidal magnetic field B_T, the poloidal magnetic field B_p generated by the current I_p, and the resulting total magnetic field B are indicated in the figure. The field lines generated by the transformer in the surrounding air are also traced.

the safety factor q_s it is necessary to use the highest value possible for the magnetic field that is compatible with the smallest value possible for the major radius of the plasma. The minimum value of the major radius R is limited by the necessity to install the windings for the transformer in the center of the torus, coaxial to the axis of symmetry. Moreover, the toroidal magnetic coil must have a cross section sufficiently large to carry the current I_c and be designed so as to resist the relevant magnetic forces that come into play. Recall that the typical values considered for

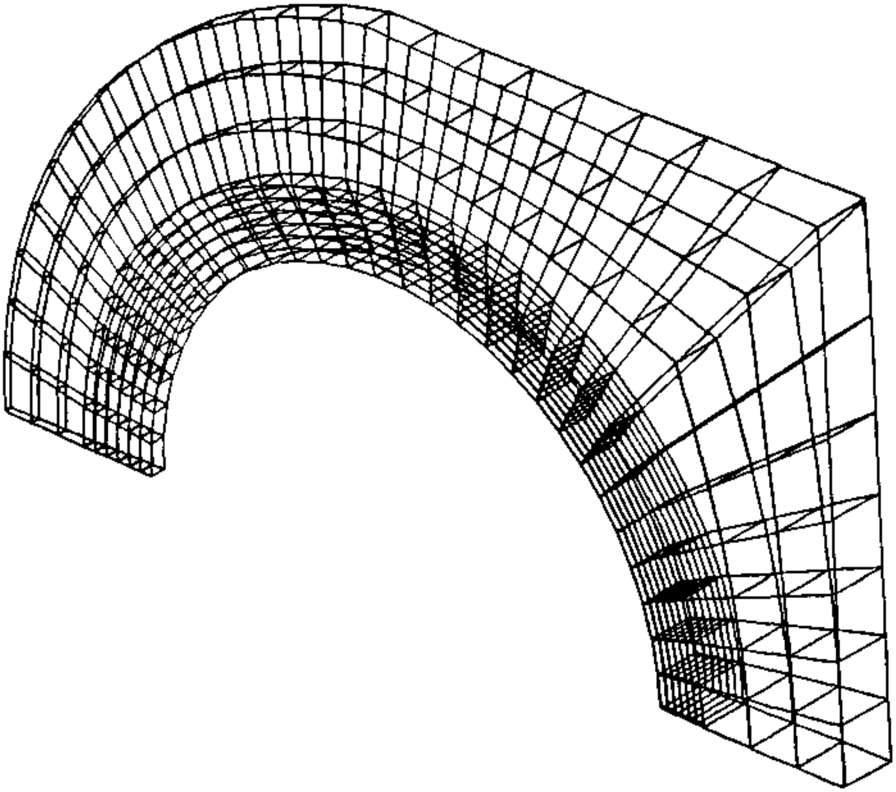

Figure 5. Three-dimensional network used for structural analysis of a toroidal magnetic element.

devices of this type are $A = 2.5$, $B_{t0} \simeq 150$ kG. This makes the field B_{ti} at the inside of the magnet, which is approximately

$$B_{ti} = B_{t0}R/(R - a) = B_{t0}A/(A - 1),$$

climb to 250 kG. Such a field is equivalent to an effective pressure of about 2500 atmospheres.

We can thus understand how the design of an Ignitor-type device might present related problems in its analysis. Such problems are confronted and resolved with finite element codes that have allowed the calculation of the effective distribution of the mechanical and thermal pressures first and then the corresponding stresses and deformations. Figure 5 illustrates a three-dimensional network used for the numerical

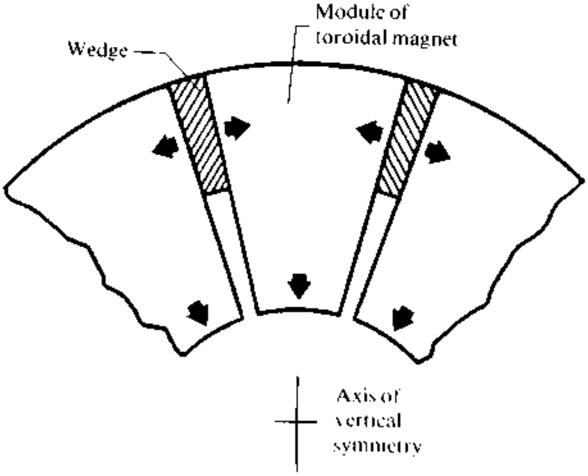

Figure 6. Solution adopted in the Frascati tokamak for the problem of implosion of the magnetic coils. The compression forces are produced by centripetal force that acts on each group of coils that make up a module of the toroidal magnet. These forces are redistributed on the outside part of the magnet, on a relatively wide surface, by means of a pair of "wedges."

calculation of the structural analysis of an element of the toroidal magnet for one of the structures considered.

The pressures to which the toroidal magnet is subjected are similar in a certain sense to those exerted on the surface of an air (or gas) chamber of elastic material, which tends to expand in the direction of either the major radius or of the minor radius. Since, moreover, the strength of the magnetic field is not uniform in the case of a toroidal magnet, the distribution of the pressures might be such as to force an implosion toward the axis of symmetry within the magnet. It is thus necessary to balance this force by connecting the copper windings that are grouped in the coils to convenient supports. These supports, which can be steel containers, must be able to prevent each coil from expanding as in the case of the air chamber, and moreover, able to balance the "wedge" effect that would produce forces of unacceptable compression in the equatorial section (see Fig. 6).

The design of the coils for the "poloidal fields," which must maintain the plasma in equilibrium and furnish the necessary flux to "fire" the current in the plasma, has particular importance. The coils must be plasma compression, is a source of additional heating, occurring in a manner analogous to that which occurs in the adiabatic compression of a gas. Other methods of additional heating, particularly the microwave-power injection method, can be applied to the Ignitor through a certain number of access ports in the plasma chamber.

These heating methods along with ohmic heating are sufficient to achieve ignition conditions. The additional power required still remains relatively modest thanks to the reduced dimensions of the device. This has been verified with numerical calculations by means of a code that can simulate the discharge in a tokamak-type device (transport code). Such a code contains a plasma model that takes into account, in the most accurate manner possible, the various processes by which the plasma loses energy.

Such processes can be divided into three categories:

Losses by Emission of Electromagnetic Radiation

Among these are processes like bremsstrahlung and cyclotron radiation, which can be accurately evaluated. The radiation loss from heavy impurities, on the other hand, is difficult to evaluate, since this depends critically on the concentration of the impurities, which is difficult to anticipate. The concentration is in turn dependent on the effects that the metallic walls of the plasma chamber are subjected to by the plasma density, the presence of density and temperature fluctuations, and the like.

"Classical Transport" Processes from the Collision between the Plasma Particles

These depend on the various collisions' frequencies and give rise to a thermal conductivity of which many aspects can be compared to those of ordinary gases. The energy loss produced by these processes tends to decrease with an increase in the temperature and values that can be well controlled.

"Anomalous" Losses

These depend on the collective modes of the plasma and not on the effects of the collisions among single particles. By collective modes are meant waves, fluctuations in density and temperature, convective cells, and the like, that can be excited spontaneously in the plasma column. For example, it is reasonable to expect that the amplitude of the density and temperature fluctuations increases with the increase in the plasma's mean temperature. Yet the effects of the collective modes are not theoretically well understood at present, and only the experimental realization of D–T plasmas in which the heating of the alpha particles plays an important role can determine if these modes constitute a serious problem.

From this point of view, the surest way to approach ignition conditions is to use plasmas with the highest possible densities. But under these conditions the plasma tends to become strongly subject to collisions and, thus, closest to regimes in which the above-mentioned "classical" processes dominate. This is, in fact, one of the guiding criteria for Ignitor experiments.

Based on calculations now completed, an Ignitor device could approach true ignition if no new mechanism of energy loss should arise in the regime between ideal and true ignition. Since nothing is known experimentally about plasmas under such conditions, and the theory is not well developed, it is obvious why there is interest in the Ignitor device to conduct experiments in such plasma regimes.

HIGH-MAGNETIC-FIELD EXPERIMENTAL DEVICES

One prototype compact experimental device is the Alcator, which was developed and built at the Massachusetts Institute of Technology in 1969.[1] Another type of compact device was built at Frascati in 1971–1972 and has the capability of achieving plasma currents of ~1 MA. A third device, the Alcator C, was built at MIT and came on line in 1979. The Alcator C is very similar to the Alcator but has larger dimensions and, like the Frascati tokamak (FT), was designed to achieve a total current of 1 MA. The characteristics of these three devices are shown in Table 1.

It is useful to recall some of the objectives achieved by the Alcator program:

The indication that energy confinement time is increased with an increase in the plasma density. The increase in plasma density is achieved by injecting neutral gas into the plasma vessel at an appropriate rate, called "gas puffing." Notwithstanding the small dimensions and low cost of the Alcator, therefore, a value of the $n\tau$ confinement parameter (the product of the density and the energy-confinement time) greater than 10^{13} sec/cm^3 was obtained for the first time. Later, in 1977, the Alcator achieved the recorded value of 3×10^{13} sec/cm^3.

The indication that the purity of the plasma (defined as the concentration of deuterium or hydrogen nuclei relative to the concentration of other ions, for example, oxygen) *increases when the density is increased.* Subsequently, the Alcator obtained record density values in the first pure plasma under conditions of thermonuclear interest.

The control of instabilities, which tend to destroy the plasma column, by means of programming the plasma density and current during the discharge. It was thus possible to achieve currents of 300 kA

Table 1. Experimental Devices with Deuterium Plasmas

Name	*Maximum magnetic field (kG)*	*Minor radius (cm)*	*Major radius (cm)*	*Maximum plasma current (MA)*	*Year device came on line*
Alcator A (U.S.)	100	10	54	0.3–0.5	1972
Frascati torus (Italy)	100	21	83	1	1977
Alcator C (U.S.)	140	17	64	1	1979

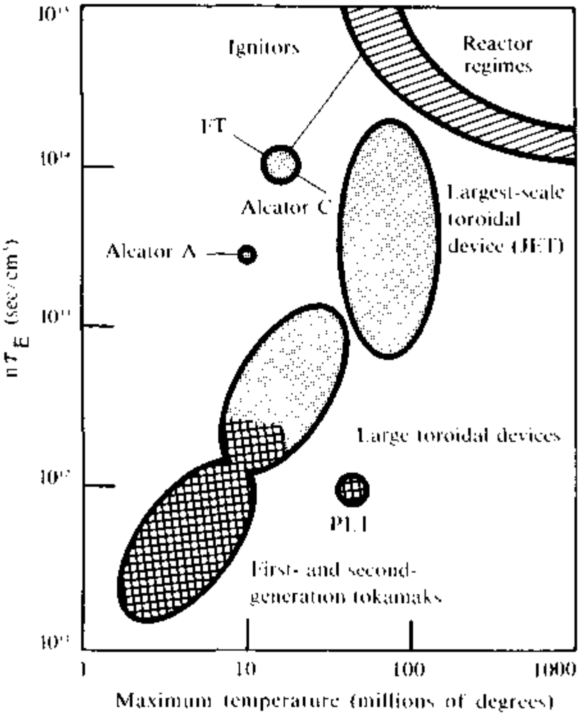

Figure 8. Toroidal experiments now in operation and projected, represented in the plane $n\tau, T$. Note the two principal lines of approach to ignition; on the left, which has Alcator A as its prototype, are compact devices characterized by high particle densities and currents; on the right is the large-volume device approach.

in a well-confined plasma with peak density of 10^{15} particles per cubic centimeter. One can observe stable discharges with a safety factor of $q_s \lesssim 2$.

The possibility of varying the plasma density by a factor of 500. One can thus realize regimes with very low densities and with a degree of collisionality so low that the rate of energy loss might be dominated by the effects of the collective modes as previously indicated. At the other extreme, that is, with the highest density values, the collisionality becomes so high that the classical process for thermal energy loss already mentioned begins to play a significant role.

What has been obtained with the Alcator has also been verified with the Frascati tokamak, which is giving optimum results, all conforming to the forecasts obtained with the transport code used by the Ignitor.

CONSIDERATIONS OF THE PHILOSOPHY UNDERLYING FUSION EXPERIMENTS BY MAGNETIC CONFINEMENT

In Figure 8 we have indicated the progress toward conditions of thermonuclear fusion, which have been realized or which are projected, developing two different lines of toroidal magnetic confinement devices. One line is represented by the compact devices, those considered in this article. These operate at a relatively high particle density and with high magnetic fields. The other line is characterized by large-volume devices, with much lower densities and magnetic fields.

This latter line is considered to be the most direct approach to a reactor that produces commercial power. In fact, the power density (measured in kilowatts per cubic centimeter) produced by a toroidal thermonuclear reactor with a relatively low particle density (e.g., 10^{13} to 10^{14} particles/cm^3) is quite low because the power flux, with respect to the first wall ("wall loading") formed by the surface of the metallic container in contact with the plasma, does not exceed a few megawatts per square meter. We recall, however, that a precise value for an economically acceptable wall loading is not well known because there is a lack of data on the behavior of the materials under the combined flux of 14-MeV neutrons and 3.5-MeV alpha particles produced in the D–T reactions, notwithstanding an intense flux of electromagnetic radiation. The general technical characteristics possessed by an "economical" reactor are also not well known, in order that the first wall have an acceptable lifetime. One cannot therefore exclude the possibility of arriving at an economical reactor following the Ignitor line. Studies concerning this are now underway.

Another factor that has determined the allocation of large-scale financial efforts for large devices to the detriment of compact devices (see Tables 2, 3, and 4) has been the impression, which arose in the early 1970s, that the scientific problems yet unsolved for achieving a demonstrably feasible thermonuclear reactor might not be of determining importance.

On the other hand, the success of the Alcator experiment, which was conceived as an experiment in basic plasma physics and not, in a certain sense, connected with the practical realization of a thermonuclear reactor, was in large part a result of the favorable effects of physical phenomena that were not well understood in 1969 when the experiment was designed. These effects are connected to the possibility of controlling plasma density by means of injection of neutral gas in the plasma vessel; with the capacity to avoid the onset of macroscopic instability that tends to disrupt the plasma column in regimes characterized by relatively high currents; with the realization of the first plasma lacking impurities in regimes of thermonuclear interest; and with the discovery of the fact that confinement time is improved with increasing plasma density.

The push toward the realization of a large-scale fusion reactor, together with the diminution of interest in research on new physical phenomena, has carried with it the decision to postpone experiments with compact devices that might burn tritium; an experiment of this type has been proposed since 1975 (see Figure 8).

The possible applications of this type of experiment concern the possibility of using it as a source of 14-MeV neutrons or as a materials-testing reactor, in which the plasma vessel itself is made of a test material.

We can now recall some inevitable disadvantages that are inherent in the approach to ignition conditions with large devices:

(1) The rate of internal ohmic heating is so low and the volume of plasma to be heated so large that this must be considered in the large designs and costly systems for additional heating of the plasma. At present, the principal system for additional heating in these devices is the injection of fast neutral atoms. For example, in the JET experiment, the temperature achieved in the plasma by ohmic heating alone cannot exceed 0.3 to 0.5 keV, and, as in the case of other large-volume experiments (Tables 2 and 4), it will be necessary to inject powers on the order of some tens of megawatts in order to raise the temperature to values of thermonuclear interest. It is not clear how this heating method can be used for large-power nuclear reactors.

As for the efficacy of injection of neutral atoms to heat the plasma, we recall that the ion temperature but not the electron temperature is brought up to 5.5 keV in the PLT device at Princeton with this heating method. This corresponds, however, to relatively low values of the particle densities and that of the $n\tau$ parameter ($n\tau \simeq 10^{12}$). No relevant

Table 2. Large-Volume Experimental Devices with Deuterium Plasmas

Name	*Maximum magnetic field (kG)*	*Minor radius (cm)*	*Major radius (cm)*	*Maximum plasma current (MA)*	*Year device came on line*
Doublet III (U.S.)	26	45 × 150	143	3	1979
JT-60 (Japan)	45	95	100	3	1981 (est.)
T-10 M (USSR)	35	80	235	1.6–1.9	1981 (est.)

Table 3. Experimental Compact Devices with Deuterium and Tritium Plasmas

Name	*Maximum magnetic field (kG)*	*Minor radius (cm)*	*Major radius (cm)*	*Maximum plasma current (MA)*	*Comment*
Ignitor	150	26–28	70	4	A device of the same type but with greater dimensions is planned for the Max Planck Institute, Garching, West Germany.
		30 32 precompression	91 precompression	3 precompression	

Table 4. Large-Volume Experimental Devices with Deuterium and Tritium Plasmas

Name	*Maximum magnetic field (kG)*	*Minor radius (cm)*	*Major radius (cm)*	*Maximum plasma current (MA)*	*Comment*
JET (Europe)	27 (34.5)	125 × 210	296	3.8 (4.8)	Estimated functioning with tritium by 1985
TFTR (U.S.)	52	85	248	2.5	Estimated functioning with tritium by 1983

increase of energy loss from instability tied to the behavior of the ions was observed; but this could have been seen theoretically. The situation instead is much more uncertain regarding the instability related to electron transport in regimes that are barely collisional.

There are theoretical indications that at low density and high temperature the losses in the "electron channel" can be noticeably greater than those encountered in plasmas obtained up to now. On the other hand, with some of the experiments performed on the PLT the electron temperature has remained rather low (2 keV). For this reason, it is safer to operate at the highest densities possible and the lowest temperatures possible, as proposed for the Ignitor.

(2) The large devices require large investments and long-term decisions. Thus every variation in a line of thought that can be undertaken on the basis of new physical information requires a long period of time for acceptance, incorporation into a project, and final realization. This is not true for compact devices, whose cost is of lower order than that of the large devices.

In conclusion, we maintain that there still exists a state of uncertainty in the understanding of many phenomena that could be important in future fusion reactors. This uncertainty means that it is not wise to invest "all or nothing" into the large devices, and not leave room for the parallel development of a line of experimental devices that, while not excessively costly, will provide information of great interest for future magnetic-confinement fusion reactors.

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Research Review

The Beginnings of a Deterministic Theory of Turbulence

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A three-day workshop on coherent structures in turbulent flow held at the Scripps Institute of Oceanography September 26 through 28, 1979, in LaJolla, California, marked an important point in the evolution of a new direction in continuum mechanics—the formalization of the idea of coherent structures as a definable body of physical phenomena.

This new direction is part of a profound change that has occurred in the past 10 years in the fields of physics that deal with continuum mechanics. In fluid mechanics, meteorology, astrogeophysics, plasma physics, and solid-state physics, there has been a new attack on the problem of dynamic evolution that not only reformulates the problems in these disciplines, but provides new avenues of attack on the frontier areas of physics generally.

Prior to 1970, the history of continuum mechanics—a part of physics identified classically with fluid mechanics—divided itself neatly into two periods, with World War II marking the division. The hydrodynamics of the first of these periods drew its empirical inspiration from the intuitively most striking feature of turbulent fluid motion: the formation of vortices, eddies, and other large-scale, coherent motions. From the Greeks, through Da Vinci, and then to Kant and Descartes, the profound impact of the observations of vortex motion on the basic ideas concerning the dynamics of continuum systems is clear. The hydrodynamics of the late 19th and early 20th century concentrated on explaining the motion and stability of these large-scale structures.[1]

By the end of the 19th century, the basic equations of fluid dynamics had been exhaustively studied by the best mathematicians but were satisfactorily solved in only a very few cases. A.B. Basset, in his classic *Treatise on Hydrodynamics* (published in 1888 and quoted by Zabusky), summarized the problem that continued to befuddle these mathematicians:

"The mathematical difficulties of solving this problem [the fluid equations] when the initial distribution of the vortices and the initial forms of their cross sections are given are very great; and it seems impossible in the present state of analysis to do more than obtain approximate solutions in certain cases."

Faced with these formidable difficulties, experimental and theoretical work gradually changed direction in the next 30 years. Based on the successes of statistical mechanics in dealing with systems that had a large number of degrees of freedom, fluid mechanists turned to statistical methods. The work of G.B. Taylor on theoretical and experimental studies of statistical properties (especially spectra and correlation functions) in the 1920s and 1930s set the stage for what became known as the modern theory of turbulence. In the last 20 years, a number of mathematically sophisticated applications of various statistical many-body approximations have been made to fluid turbulence, primarily under the influence of Robert Kraichman, and these theories have dominated the field of fluid mechanics.[2]

All these statistical theories are based on the observation that turbulence is a disordered flow of fluid, whose only predictable features are various means, moments, and correlation functions. The pervasive lesson of Boltzmann's kinetic theory of gases dominated the thinking in fluid dynamics. The greater the number of degrees of freedom, the greater the chance of disordered motion in the small and the greater the chance of statistically smooth behavior in the large. Both experimental and theoretical work was (and still is, to a large extent) motivated by the conviction that the classical quest for causality and predictability in complex systems had to be sacrificed for statistics. As Orzag put it:

"[A second] important characteristic of turbulent flows is their apparent randomness and instability to small perturbations. Two turbulent flows that are at some time nearly identical in detail do not remain nearly identical on the time scales of dynamical interest. This property of turbulent flows may be used to give a quantitative definition of turbulence. Also, instability of turbulent motion is related to the limited 'predictability' of atmospheric motions.

"While the details of fully developed turbulent motions are extremely sensitive to triggering disturbances, average propertites are not. Otherwise there would be little significance in the averages. On the other hand, transition flows (which occur naturally at Reynolds numbers several times critical) have statistical properties which are sensitive to the nature of the perturbations. The idea that fully developed turbulent flows are extremely sensitive to small perturbations but have statistical properties that are insensitive to perturbations is of central importance. . . ."

There was just one persistent problem with these statistical theories: The quantities they could predict tended to be the least interesting, and, in general, they were unable to explain turbulent phenomena at all.[3] However, this fact is recognized only by a relatively small number of fluid dynamicists. Norman Zabusky fired one of the opening shots in a long-overdue reassessment of statistical approaches to physics:

"In the last decade we have experienced a conceptual shift in our view of turbulence. For flows with strong velocity shears, near boundaries, density gradients, magnetic fields or other organizing characteristics, many now feel that the spectral or wave-number space description has inhibited fundamental progress.

"The next 'El Dorado' lies in the mathematical understanding of coherent structures in weakly dissipative fluids: the formation, evolution, and interaction of metastable, vortex-like solutions of nonlinear, partial differential equations."

John Laufer makes the same point from the point of view of an experimentalist in his paper:

"In the past 10 years two important observations were reported that had a significant impact on subsequent turbulence research. Ironically, these were made not by sophisticated electronics instrumentation but visually with rather simple optical techniques. The essence of these observations was the discovery that turbulent flows of simple geometry are not so chaotic as had been previously assumed: There is some order in the motion with an observable chain of events recurring randomly with a statistically definable mean period. This surprising result encouraged researchers to reexamine the line of inquiry for designing their experiments, and they began seriously questioning the relevance of some of the statistical quantities they had been measuring. It was soon realized, for instance, that retaining some phase information in the statistics and obtaining more detailed spatial information are essential for a quantitative explanation of the visual observations."

The discontent with statistical, nonpredictive approaches to fluid dynamics has come to a head in the past several years, with the coalescence of empirical data (especially in superfluids, boundary layer mixing experiments, and measurements of wakes), numerical simulations,[4] and theoretical work. In all these areas it became obvious that statistical averaging was in fact destroying the most interesting and important phenomena in turbulence--the formation, dynamics, and persistence of large-scale structure.

THE LAJOLLA WORKSHOP

The workshop at LaJolla brought together 50 of the pioneers in this new direction in continuum mechanics for a discussion of the outstanding problems posed by their new approach. The result of the discussions and research presented at the workshop is the outline of a *deterministic theory of turbulence*. It is universally recognized that a negative critique of statistical hydrodynamics, however well-deserved, is not enough; a new theory that is capable of more than just describing coherent phenomena must be formulated. The first steps in this formulation were the subject of the LaJolla meeting.

G. Brown and A. Roshko presented a series of papers at the conference summarizing their theoretical and experimental work. They reported on their now-confirmed hypothesis of the existence of a predictable, regular interaction between vortices, a process they call *pairing*. After examination of many hundreds of movie films of interaction in a turbulent boundary layer (between two oppositely directed fluid flows), they have observed that the vortex dynamics are determined by a hierarchical pairwise coalescence of the vortices in the boundary layer.

The problem is visually very striking. Two fluids of different optical properties are shot into a chamber at different velocities. A movie camera then follows the fluid motion in a frame of reference such that the fluids are moving with equal, opposing velocities (that is, the camera moves at the mean velocity of the two gases, so that the slower one seems to be moving backward, and the faster gas forward—at the same speed but in opposite directions).

The resultant flow pattern in its initial stages (shown in Fig. 1), is the classical von Karman vortex street. In a 1911 paper, von Karman derived a simple stability criterion for the configuration of alternating opposite-signed vortices that results from the counterflowing fluids. Von Karman's analysis showed that stability of a vortex street occurred only for a specific ratio of vertical to horizontal spacing of the vortex centers; there has been no successful nonlinear treatment of von Karman's original linear stability analysis.

In spite of the analytical intractibility of the problem, it has attracted extensive experimental attention. One of the most striking of these "experiments" was performed at Bell Laboratories in the late 1960s and early 1970s by Norman Zabrusky, F. Tappert, G. Deem, and G. Hardin. In an extensive series of computer solutions to the Navier–Stokes equation, they examined the stability of the interface between counterflowing fluids—the same problem considered by von Karman and Brown–Roshko. Their solutions reproduced the von Karman instability in the early stages of the simulation, as the interface between the fluids "rolled up" into the set of counterrotating vortices in the spacing predicted by von Karman.

The vortex street then proceeds to undergo an evolution that was totally unexpected—it collapses into a jumble of tangled vortex lines that give the appearance of being totally random. A chaotic and seemingly "turbulent" (in the statistical sense) period ensues in the fluid. In the final steps of the computer run, however, a new vortex street begins to coalesce out of the chaos, a vortex street with a scalelength roughly twice that of the original. This striking observation remained only speculative, however, since the simulation broke off before this second vortex street itself had stabilized.

In the laboratory experiment by Brown and Roshko, the usual experimental result is shown in Figure 1, where the regular pattern of vortex motion is obvious. But Brown and Roshko also observed a chain of increasingly complex motions whose regularity was hidden until the formulation of the idea of pairing.

The results that they summarized at the LaJolla meeting showed the following sequence of events.

(1) The initial vortex pattern (like that shown in Fig. 1) breaks up after a distance determined by the physical properties (such as viscosity) of the fluid and the velocity of the flow. There is a region of apparent random and quite violent turbulence.

(2) Out of this turbulence, a new, highly ordered vortex flow is formed, with a scalelength roughly twice that of the original flow.

(3) In cases where the experiment can be followed for long times in large chambers, the process seems to be repeated.

After the careful study of hundreds of such experiments using different fluids and different diagnostics (shadowgraph techniques), these researchers found that the vortices in the original boundary layer pair up, with neighboring, like-signed vortices coalescing to form a larger vortex with the same sense of rotation. The area of seemingly random motion between the two well-organized vortex flows is a result

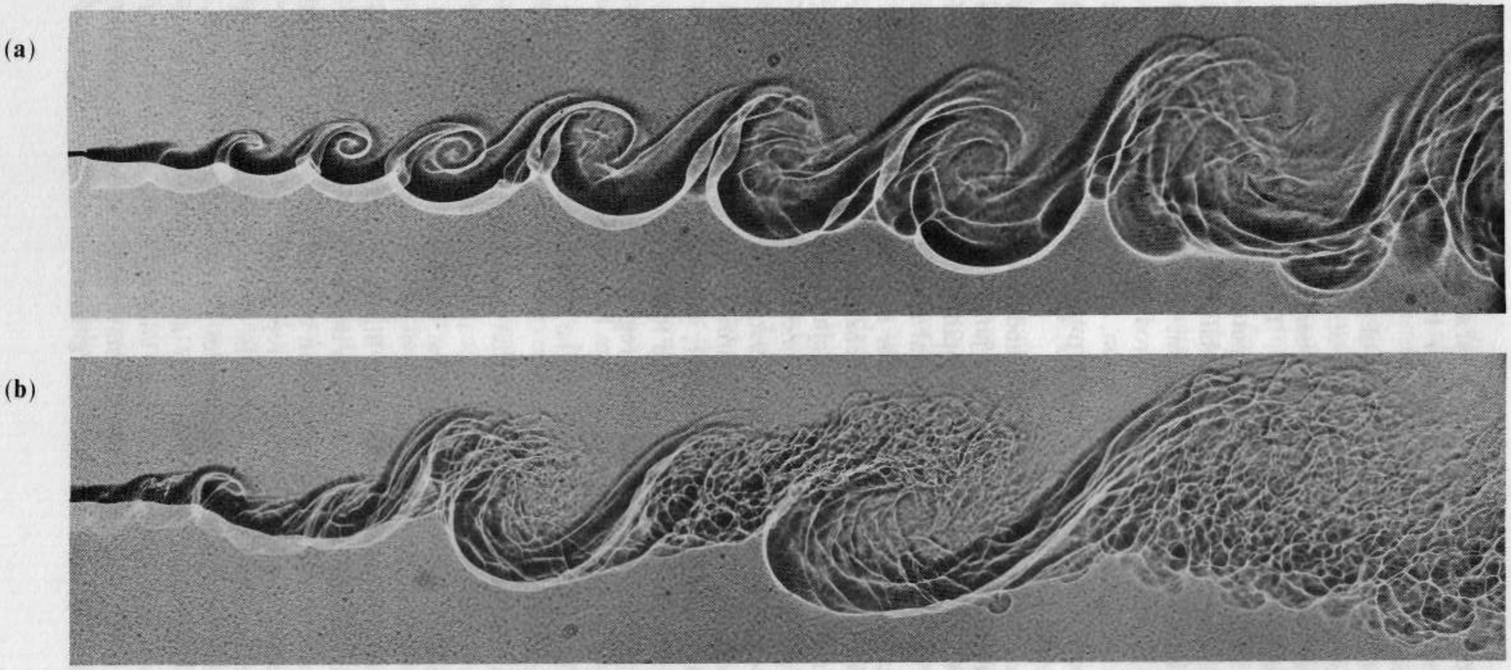
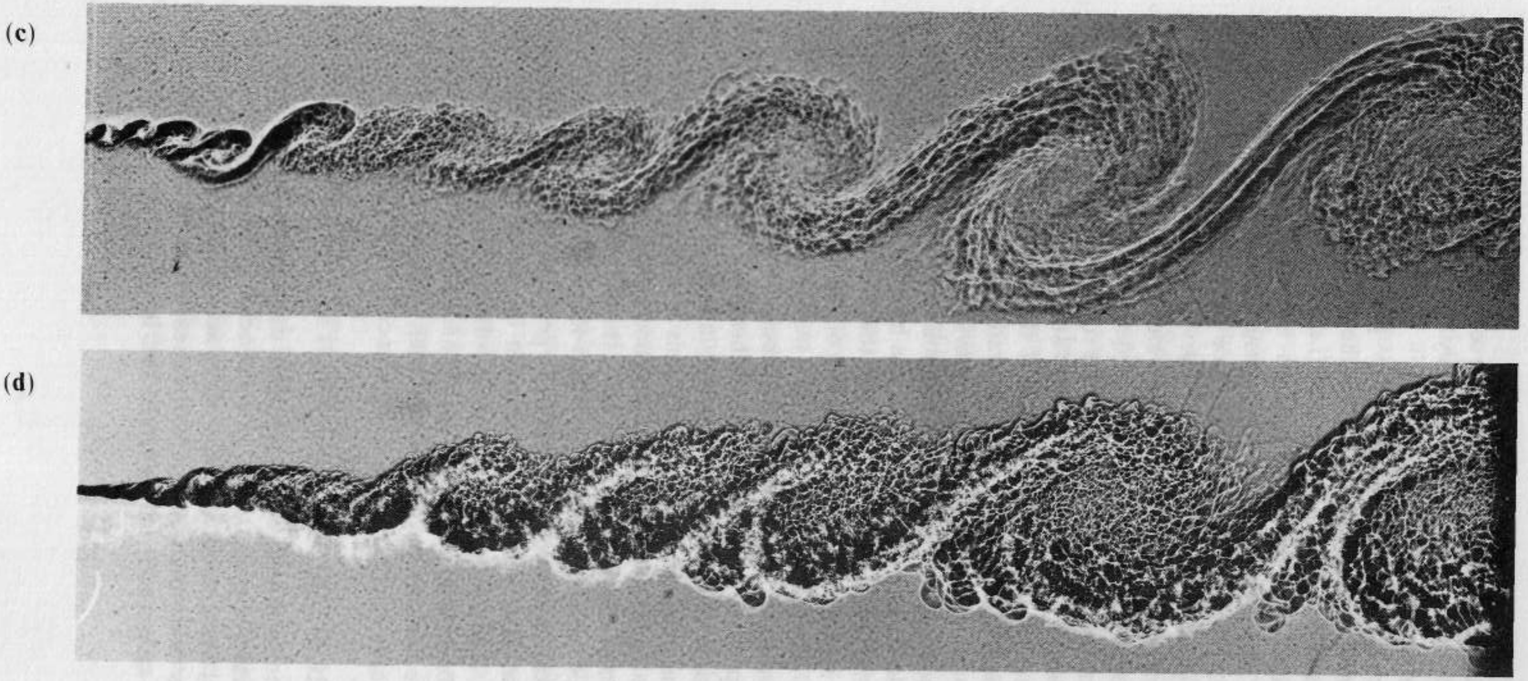


Figure 1. This series of flow visualizations by Brown, Roshko, and their students M. Rebollo and J. H. Konrad, demonstrates in a dramatic way their hypothesis concerning the relation between turbulence and large-scale structure. Conventional thinking has been that turbulence occurs as the end state of flows with high Reynolds number (a dimensionless constant proportional to average velocity divided by viscosity). The usual analysis of the Navier–Stokes equation shows that the asymptotic state of high-Reynolds-number flows is chaotic, random, small-scale motion. In this series of photographs, we see a mixing shear layer formed by counterstreaming jets of helium and nitrogen. The interface at which the gases are in contact "winds up" in the vortical pattern shown. By increasing the relative



velocity of the two gases, the Reynolds number is also increased. The Reynolds number in Figure 1(a) is low, giving rise to a smooth, vortical interface. But, as shown in (b) and (c), as the speed of the gas is increased, the flow becomes more and more dominated by small-scale motion. The last picture (d) is remarkable: here, in a flow almost completely dominated by small-scale turbulence (in accordance with conventional expectations), there is large-scale coherent structure! This *large-scale structure*, although *not made up of the small-scale motion*, determines the long-range correlations and behavior of the flow, in a way inexplicable using conventional mechanisms. (Photographs courtesy of Anatol Roshko, Professor of Aeronautics, California Institute of Technology.)

of a complicated and not-yet-understood coalescence. It is a surprising indictment of statistical methods that they show this transition area to be random in the strictest sense.

A number of the properties of this pairing process have been elaborated in some detail and applied to the study of other properties of turbulent flow. The generation of noise (i.e., acoustical radiation from turbulent flows) has been hypothesized to result from this pairing interaction. This regularity, previously unsuspected but now thoroughly documented, is of the same significance as Kepler's three laws of planetary dynamics. It demonstrates the underlying lawfulness of a large class of phenomena that had previously been assumed by most physicists to be inherently random and only describable in statistical terms. It is an astounding insight into the dynamics of collective vortex interaction and promises a truly deterministic theory of turbulence.

Roshko, in a talk on the third day of the workshop, described the methodological approach that has motivated his work. He noted that the primary intuition that has guided previous work on turbulence has been an hypothesis concerning its asymptotic disorder. If one waits long enough, or looks at a fluid with velocities high enough (high Reynolds numbers), this hypothesis predicts that the flow will become random. This belief is, perhaps, the quintessence of the statistical approach to turbulence—the belief that it is merely a matter of waiting long enough for the actual statistical nature of the turbulence to show itself.

Roshko pointed out that this hypothesis has never been proved mathematically or empirically, and that, in fact, in many cases experimental counterdemonstrations upheld this hypothesis of disorder only because the diagnostics used were too crude to detect order in the flow. He showed a remarkable set of slides of the same flows using different photographic techniques, some of which looked totally random while others (of the same flow) showed a large-scale order that had been "washed out" by the lack of resolution in the cruder photographic techniques. Roshko summarized his hypothesis that large-scale structure is the basic property of turbulence, and that the two are inseparable. To the idea of asymptotic disorder, Roshko counterposed the dictum: "Large-scale structure *is* turbulence."

NEW FRONTIERS

The workshop was pervaded by a sense of excitement and discovery rare in most areas of physics today. In a real sense, this group represents a minority faction in the community of physicists at large. The hegemonic conceptions and intuitions guiding research in physics today find large-scale coherent structure irrelevant. In one of the final presentations, A. Ingersoll, from the Jet Propulsion Laboratory in California, presented some results from the Jupiter (Voyager) satellites and their implications for continuum mechanics. His talk captured the sense of discovery and new physics that characterized the meeting.

Ingersoll showed a spectacular series of slides and movies of Jupiter's atmosphere as observed by Voyager 1. The simple empirics of the planet's atmosphere are astounding: Jupiter's atmosphere is very large (with linear scalelengths about eleven times larger than the Earth), rotates faster (with a 9-hour period), and is characterized by very-large-scale vortex structures (some the size of the Earth!), which are very stable (the Great Red Spot was first observed 200 years ago) and are imbedded in a highly regular zonal flow. Figure 2 shows a cylindrical projection of the whole surface of Jupiter—the zonal bands have counterrotating flow in an almost textbook example of "trade winds." Ingersoll showed slide after slide of similar phenomena.

Figure 3 shows structures in the wake of the Red Spot; Figure 4 shows the regular atmospheric waves in a polar projection of Jupiter; and Figure 5 shows an incredible picture of a *three-dimensional helical wake* in the atmosphere.

Perhaps most surprising was the movie that Ingersoll showed of the atmospheric motions taken over a several-week period. By assembling pictures of the planet taken every 9 hours by the satellite as it approached Jupiter, the "weather" on the planet was recorded in a motion picture that forces even the most jaded observer to conclude that something extraordinary is going on—the evidence for ordered, large-scale motion is unavoidable.

As Ingersoll and others have pointed out, the evidence from Jupiter has very far-reaching implications for fluid mechanics. Here is a fluid with an astronomical Reynolds number (on the order of 10^5 to 10^6). In this range it is almost universally assumed that all fluid flow will be randomized and totally disordered. Yet, here is a large example of the opposite fluid evolution. Ingersoll concluded his talk with a refreshing response to a quite defensive question from a physicist threatened by these implications: "Everything about Jupiter is astounding."

BROADER IMPLICATIONS

The most recent theoretical work on the formation of large-scale structure in turbulence (see *Fusion*, Dec. 1979, p. 24) is clearly divisible in terms of the approach taken to the initial formation of vortex motion. The most common approach, a direct continuation of von Karman's original work, *assumes* from the beginning that the vortices exist, and

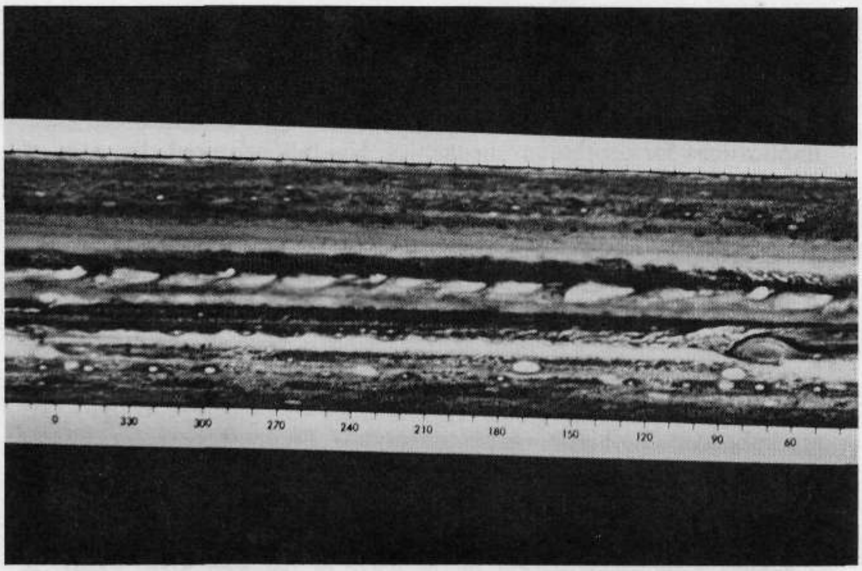


Figure 2. Cylindrical projection of the Jovian atmosphere: This montage of photographs from the Voyager satellite shows dramatically the large-scale wavelike features on the planet. The regular spacing of the cyclonic and anticyclonic storms, the fixed-wavelength ripples in the latitudinal winds, and the latitudinally stratified flow are all striking. A length scale can be estimated by comparing the size of the Earth to the Great Red Spot, whose diameter is about three times that of the Earth.

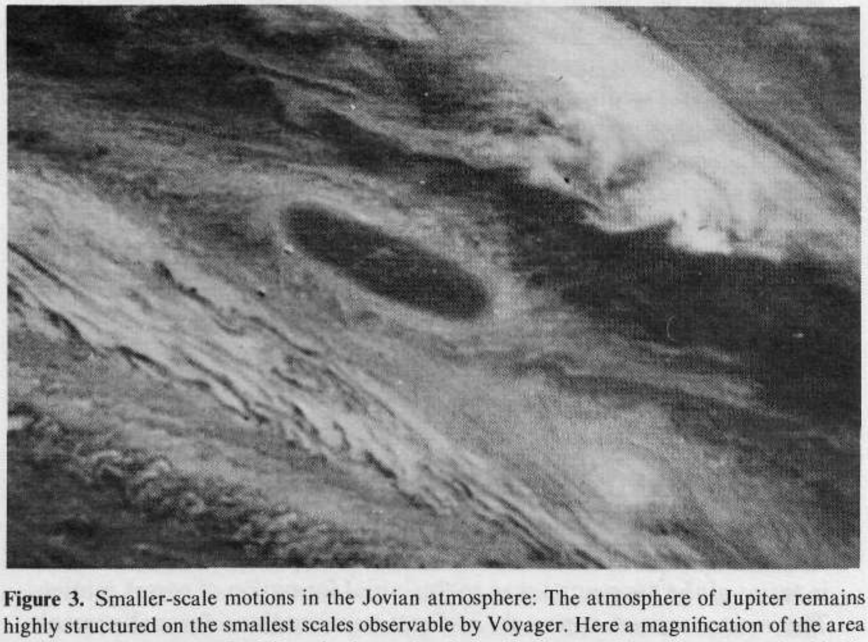


Figure 3. Smaller-scale motions in the Jovian atmosphere: The atmosphere of Jupiter remains highly structured on the smallest scales observable by Voyager. Here a magnification of the area southwest of the Great Red Spot is shown, in the wake generated by the storm. The highly convoluted flow is a beautiful example of structured turbulent flow in a high-Reynolds-number fluid.

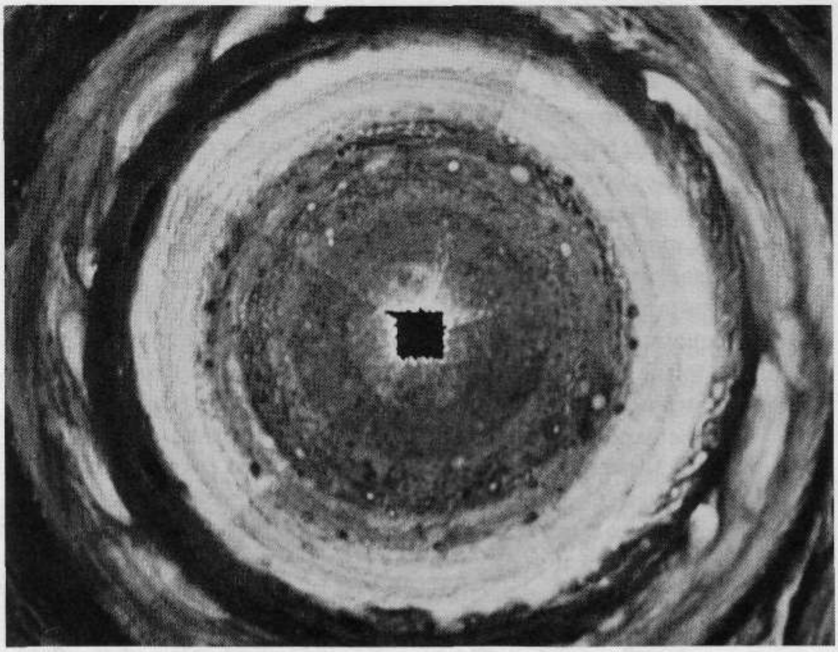


Figure 4. Polar projection of the Jovian atmosphere: This montge of satellite photographs shows the south polar region of Jupiter (the central dark area was not photographed). The regular wave patterns seen in the cylindrical projection of Figure 2 are here evident as the result of a constraint introduced by the spherical shape of the planet: the waves must fit an integral number of wavelengths around any latitude. The storms in this photograph are also arranged at strikingly regular intervals around the pole.

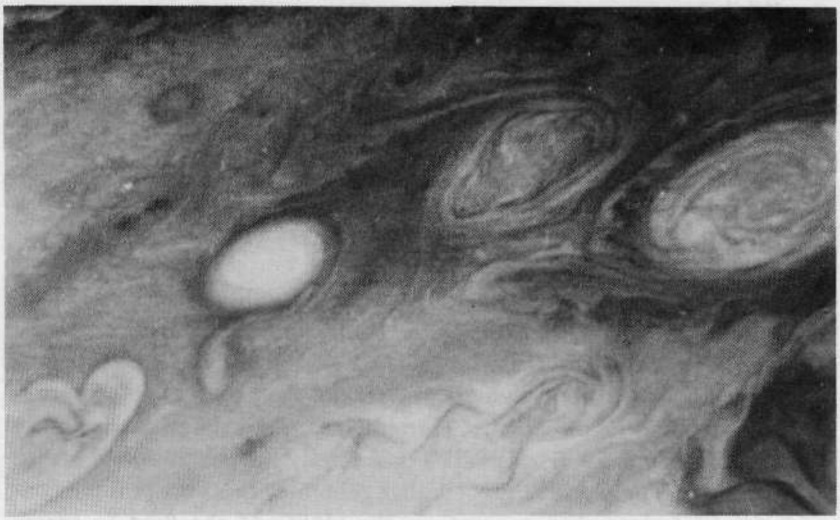


Figure 5. A three-dimensional helical wake of the Great Red Spot: In one of the most remarkable pictures of the Jovian atmosphere, the Great Red Spot has produced a wake that is clearly in the form of a three-dimensional helix. The atmosphere flowing around the storm of the Great Red Spot exhibits the same geometrical structure as terrestrial laboratory fluids but at thousands of times larger scale lengths.

then uses the energy and potential functions for a *given vortex array* to determine the stability and linear dynamics of that array.[5]

These studies provide an illuminating description of many aspects of vortex pairing phenomena, but can hardly be called an explanation. The more basic questions of the energetics of formation of the vortex array itself are unapproachable using this technique.

A second approach has been the numerical solution of the Navier–Stokes equation in an attempt to map out the phase space of real solutions to the equations of fluid turbulence. Many of these solutions show the formation of vortex structures and reproduce in quite astounding ways the experimental results of Brown and Roshko. However, as intriguing as these studies are, they have yet to shed fundamental light on the questions of large-scale structure. Some new results on the spontaneous appearance of singularities in fluid flow are the first results from numerical work that promise this sort of dramatic new insight.

A third approach developed by researchers at the Courant Institute in New York City has been a topological approach to the real-space structure of vortex motion. This group, using a reduction of the fluid equations, has shown how singularities can appear in finite time in a fluid flow, specifically in the form of apparent violations of Helmholtz's theorem in an ideal fluid. These results[6] provide the first analytic demonstration of the appearance of singular, topology-changing phenomena in an ideal fluid.

While by no means a majority view at the meeting, there is a growing sense among workers in the field that present theoretical methods are incurably flawed. The most interesting evidence, discussed informally at the conference, are these solutions that, while starting from physically reasonable initial conditions, give rise to singular flows in a *finite time*. That is to say, the Navier–Stokes equation, which should describe fluid motion if Newton's laws apply to the fluid, breaks down after a finite period of time evolution—a breakdown evidenced by the appearance of infinite velocity gradients.

There has been some speculation on the possibility of such singularities since the early 1970s, but it was only recently that quantitative evidence for such a singularity became available. The first evidence came in theoretical and numerical work by the group at the Courant Institute under the direction of Harold Grad. They describe the ability of a fluid to "violate" Helmholtz's theorem by the generation of closed streamlines. The physical process that generates this apparent nonconservation of streamline topology is the ability of the fluid to create singularities spontaneously in finite time—these singularities change the connectedness of the fluid volume and so allow a "violation" of Helmholtz's theorem.

More recently, Morf, Orszag, and Frisch did a series of computer calculations that point in the same direction:

"For boundary-free flow, the Kelvin and Helmholtz theorems imply that an initially smooth, inviscid flow remains smooth so long as vortex lines are stretched only a finite amount. Indeed, the restriction of the flow to two space dimensions precludes vortex-line stretching so global regularity follows. However, in three dimensions, vortex lines can twist, tangle, turn, and stretch. It is conceivable that flow velocities remain bounded and, still, a singularity of the flow appears spontaneously after a finite time in the interior of the flow. Segments of vortex lines could develop infinite length by becoming intricately wound up and twisted without the end points of the segment being separated by an infinite distance."

The evidence presented in both these papers, and discussed at the LaJolla conference, raises a series of deeper questions about the implications of the experimental evidence of the central role played by coherent structure, and the theoretical difficulties:

(1) Are these singularities "real?" That is, are there physical phenomena (intermittency, etc.) to which these singularities correspond, in the same sense that the singularities in the equations of gas dynamics indicate the onset of a new set of laws governing shock waves?

(2) If this conjecture is correct, then there must be something like a "characteristic" equation—as there is for shock waves—that provides an invariant for the overall evolution of turbulence. This invariant would provide the deeper explanation for the appearance and dynamics of vortex phenomena as well as for the ubiquitous nature of coherent structure in turbulent fluids.

(3) What is the broader implication of an invariant specification of the qualitative nature of time evolution in a nonlinear system? That is, such an invariant would describe the seemingly nonentropic nature of structure formation in "geometrical" terms. Can the same be done for other continuum systems?

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2. For a comprehensive account of the modern statistical theory of turbulence, see Orszag's *Lecture Notes on the Statistical Theory of Turbulence*.
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5. See J.H. Ferziger, *Phys. Fluids* **23**, 1 (1980), for a discussion.
6. H. Grad, "Reconnection of Magnetic Field Lines in an Ideal Fluid," unpublished report.